**1** Given that $a \times 60 = b$ work out the value of $\frac{4b}{a}$

**[2 marks]**

$b = 60a$

$\frac{4(60a)}{a}$ 1 $= 240$ 1

Answer 240

**2** The $n$th term of a sequence is $\dfrac{n(n-4)}{\sqrt{n+3}}$

Work out the sum of the 1st and 6th terms.

**[3 marks]**

1st term : $\dfrac{1(1-4)}{\sqrt{1+3}} = \dfrac{-3}{\sqrt{4}} = \dfrac{-3}{2}$ 1

6th term : $\dfrac{6(6-4)}{\sqrt{6+3}} = \dfrac{12}{\sqrt{9}} = \dfrac{12}{3} = 4$ 1

Sum : $-1.5 + 4 = 2.5$ 1

Answer 2.5

**3** A curve has the equation $y = x^2 - 6x + 17$

The turning point of the curve is at $(a, 8)$

**3 (a)** By completing the square, or otherwise, work out the value of $a$.

**[2 marks]**

$y = (x-3)^2 - 9 + 17$

$= (x-3)^2 + 8$ 1

Answer 3 1

**3 (b)** The turning point of the curve $y = x^2 + 4x + b$ also has $y$-coordinate 8

Work out the value of $b$.

**[2 marks]**

$y = (x+2)^2 - 4 + b$ 1

$-4 + b = 8$

$b = 12$ 1

Answer 12

4 $f(x) = 3x^2 - 4x + 8$ for all values of $x$

Jenny says,

"f(10) must equal 2 × f(5), because 10 is 2 × 5"

Is Jenny correct?

Show working to support your answer.

**[2 marks]**

$f(10) = 3(10)^2 - 4(10) + 8$

$= 300 - 40 + 8$

$= 268$

$f(5) = 3(5)^2 - 4(5) + 8$ 1

$= 75 - 20 + 8$

$= 63$

$2 \times f(5) = 2 \times 63 = 126$ 1

No. Jenny is wrong.

**5** $f(x) = 2x - 3$ and $g(x) = x^2$

Show that $f^{-1}(55) = fg(4)$

**[4 marks]**

let $f(x) = y$

$y = 2x - 3$

$y + 3 = 2x$ (1)

$x = \frac{y+3}{2}$

$f^{-1}(x) = \frac{x+3}{2}$ (1)

$f^{-1}(55) = \frac{55+3}{2}$

$= \frac{58}{2}$

$= 29$ (1)

$fg(x) = 2(x^2) - 3$

$= 2x^2 - 3$ (1)

$fg(4) = 2(4)^2 - 3$

$= 2(16) - 3$

$= 32 - 3$

$= 29$

**6 (a)** $f(x) = cx + d$

$f(4) = 7$

$f(10) = 22$

Work out the values of $c$ and $d$.

**[3 marks]**

$f(4) = 7 = 4c + d \quad - (1)$ (1)

$f(10) = 22 = 10c + d \quad - (2)$

$(2) - (1)$ :

$22 - 7 = 10c - 4c + d - d$ (1)

$15 = 6c$

$c = \frac{15}{6} = 2.5$

$7 = 4(2.5) + d$

$d = 7 - 10$

$= -3$ (1)

$c =$ 2.5 $\quad d =$ -3

**7** $L$ is directly proportional to $D^2$

$L = 85$ when $D = 10$

**7 (a)** Work out an equation connecting $L$ and $D$.

**[3 marks]**

$L = kD^2$ 1

$85 = k(10)^2$

$k = \frac{85}{100} = 0.85$ 1

$L = 0.85D^2$ 1

Answer $L = 0.85D^2$

**7 (b)** Work out the value of $L$ when $D = 5$

**[2 marks]**

$L = 0.85(5)^2$ 1

$= 0.85 \times 25$

$= 21.25$ 1

Answer $21.25$

**8** $\frac{a}{b} = 3c$

$\frac{b}{c} = 2$

Work out the value of $a$ when $c = 8$

**[3 marks]**

$b = 2c$ 1

$\frac{a}{2c} = 3c$

$a = 6c^2$ 1

$= 6(8)^2 = 6(64) = 384$ 1

Answer 384

**9** The equation of a curve is $y = 16^x$ $16^2 = 256$

**9 (a)** A different point on the curve has $y$-coordinate $\frac{1}{16}$

Work out the $x$-coordinate.

**[1 mark]**

$\frac{1}{16} = 16^x$

$x = -1$

Answer $-1$ 1

**10** $f(x) = 2x + 5$

Show that $3f(x) - 12f^{-1}(x)$ simplifies to an integer.

**[4 marks]**

let $f(x) = y$

$y = 2x + 5$

$y - 5 = 2x$ (1)

$x = \frac{y-5}{2}$

$f^{-1}(x) = \frac{x-5}{2}$ (1)

$\therefore 3(2x+5) - \cancel{12}^{6}\left(\frac{x-5}{\cancel{2}_1}\right)$ (1)

$= 6x + 15 - 6x + 30$

$= 45$ (1)

**11** Here are two simultaneous equations.

$y = x^2 + 7x - c$

and

$y = 3x + d$

There is a solution when $x = 5$

Work out the value of $c + d$

**[3 marks]**

$x^2 + 7x - c = 3x + d$ (1)

$x^2 + 7x - 3x = c + d$ (1)

$x^2 + 4x = c + d$

$(5)^2 + 4(5) = c + d$

$25 + 20 = c + d$

$45 = c + d$ (1)

Answer 45

**12 (a)** $f(x) = kx^2$ where $k$ is a constant.

Kai says that $\frac{f(6)}{f(2)}$ is equal to f(3) because $\frac{6}{2} = 3$

Is he correct?

Show working to support your answer.

**[2 marks]**

$f(6) = k(6)^2 = 36k$

$f(2) = k(2)^2 = 4k$

$f(3) = k(3)^2 = 9k$

$\frac{f(6)}{f(2)} = \frac{36k}{4k} = 9$ (1) $f(3) = 9k$ (1)

$\therefore$ No. Kai is not correct.

**13** $f(x) = x^2 + 6x$

$g(x) = 2x + 4$

**13 (a)** Solve fg($x$) = –5

**[3 marks]**

$4x^2 + 28x + 40 = -5$

$4x^2 + 28x + 45 = 0$ (1)

$x = \dfrac{-28 \pm \sqrt{28^2 - 4(4)(45)}}{2(4)}$ (1)

$= \dfrac{-28 \pm \sqrt{64}}{8}$

$= \dfrac{-28 \pm 8}{8} = \dfrac{-20}{8}$ or $-\dfrac{36}{8}$

$= -2.5$ or $-4.5$

Answer $-2.5$ and $-4.5$ (1)

**14** $f(x) = \dfrac{3x+9}{5}$ and $g(x) = 6x - 1$

**14 (a)** Show that gf(2) is an integer.

**[2 marks]**

$gf(x) = 6\dfrac{(3x+9)}{5} - 1$ (1)

$= \dfrac{18x+54}{5} - 1$

$gf(2) = \dfrac{18(2)+54}{5} - 1$

$= \dfrac{36+54}{5} - 1$

$= 18 - 1 = 17$ (1)

**14 (b)** Show that $f^{-1}(8)$ is **not** an integer.

**[2 marks]**

let $f(x) = \dfrac{3x+9}{5}$

$y = \dfrac{3x+9}{5}$

$5y = 3x+9$

$5y - 9 = 3x$

$x = \dfrac{5y-9}{3}$

$f^{-1}(x) = \dfrac{5x-9}{3}$ (1) $= \dfrac{5(8)-9}{3} = \dfrac{31}{3} = 10.3$ (1)

**15** $H$ is inversely proportional to the cube root of $L$.

$H = 7$ when $L = 64$

**15 (a)** Work out the value of $H$ when $L = 2744$

**[2 marks]**

$H = \frac{28}{\sqrt[3]{2744}}$ 1

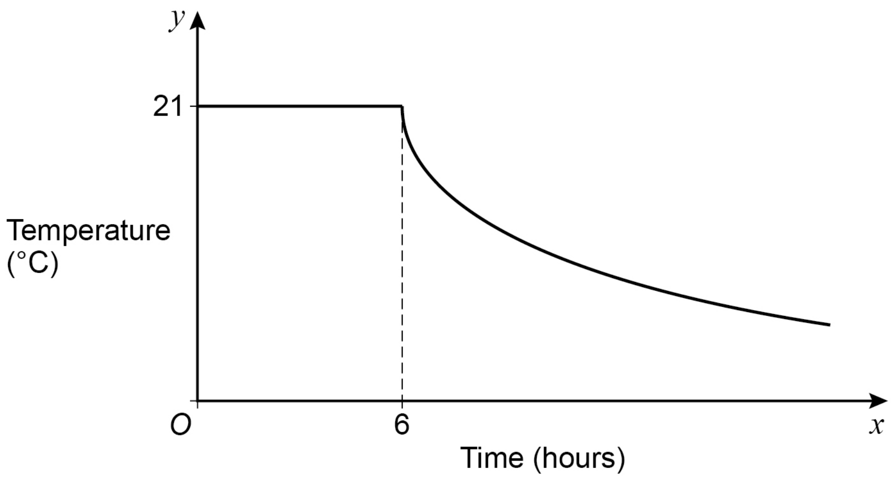
$H = \frac{28}{14} = 2$ 1

$H =$ 2

**16** A room is kept at a constant temperature of 21°C for 6 hours.

The heating is then turned off and the room begins to cool.

Here is a sketch graph showing the temperature, $y$°C, of the room at time $x$ hours.




**16 (a)** Assume the equation of the curved part is $y = \frac{k}{x}$ where $k$ is a constant.

Work out the value of $y$ when $x = 12$

**[2 marks]**

when $x = 6, y = 21$ : $21 = \frac{k}{6}$

$k = 21(6) = 126$ ✓ ①

when $x = 12$, $y = \frac{126}{12} = 10.5$

$y =$ 10.5 ✓ ①

**16 (b)** In fact,

the equation of the curved part is $y = A \times \left(\frac{1}{3}\right)^{\frac{1}{6}x}$ where $A$ is a **different** constant.

How does this affect the value of $y$ when $x = 12$ ?

Tick **one** box.

You **must** show working to support your answer.

**[2 marks]**

☐ The value of $y$ is greater than the answer to part (a).

☑ The value of $y$ is less than the answer to part (a).

✓ ①

☐ The value of $y$ is the same as the answer to part (a).

when $x = 6$, $y = 21$ : $21 = A \times \left(\frac{1}{3}\right)^{\frac{1}{6}(6)}$

$A = 21 \times 3 = 63$ ✓ ①

when $x = 12$ : $y = 63 \times \left(\frac{1}{3}\right)^{\frac{1}{6}(12)}$

$y = 63\left(\frac{1}{9}\right)$

$= 7$